(either Bose *or* Fermi), taken together with Eq. (7), is therefore

$$F(\theta,\phi)=F(\pi-\theta,\pi+\phi)=F(\pi-\theta,\pi-\phi). \qquad (8)$$

Thus normal statistics *requires* the density of events at (θ,ϕ) to be equal to the density of events at $(\pi-\theta,\pi-\phi)$. To achieve greater statistics in an experiment one would integrate the distribution F over the azimuthal angle ϕ from 0 to π. Calling this distribution $P(\theta)$, we have

$$P(\theta)=P(\pi-\theta). \qquad (9)$$

We note that: (a) if K mesons do not obey normal statistics, Eq. (9) need not be satisfied, since a given isotopic spin component of the two K meson system could enter in conjunction with terms both even and odd under $\hat{Q}\rightarrow-\hat{Q}$; (b) if the K^+ and K^0 were nonidentical isotopic singlets[5] Eq. (9) need not be satisfied. Also, we note that the restriction in Eq. (9) is more general than certain simple production mechanisms for reaction (3) would imply. For example, if the mechanism were antilambda annihilation on the proton into a forward produced virtual K^+ which then interacted with the neutron via K^0 exchange to produce the final K^+-K^0 system, the distribution F would simply tend to be independent of ϕ, (neglecting internal deuteron motion) by the argument of Treiman and Yang.[6] However, the dominance of K^0 exchange in producing a single resonant K^+-K^0 system with angular momentum L would lead to $F\propto|P_L(\cos\theta)|^2$ and hence Eq. (9) would be satisfied.

The argument for reactions (1) and (2) parallels that given above and the test of normal statistics is given, for the corresponding distributions, by Eqs. (8) and (9).

I would like to thank Professor C. N. Yang and Professor Richard Plano for considering this test and for very helpful discussions. I also thank Professor Georges Temmer for his helpful comments.

[5] A. Pais, Phys. Rev. **112**, 624 (1958).

[6] S. B. Treiman and C. N. Yang, Phys. Rev. Letters **8**, 140 (1962).

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Monte Carlo Calculations Applied to a Determination of Four-Momentum Transfer in Ultra-High Energy Interactions*

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Four-momentum transfer between two groups of particles produced in ultra-high energy interactions is studied employing 3000-BeV Monte Carlo jets generated for an earlier paper from information based largely on experimentally determined average properties of 10^{12}-eV jets. First, a parameter introduced by Hasegawa and Yokoi for measuring a deliberate underestimate of this four-momentum transfer is calculated using the Monte Carlo jets. In general, the results agree with the experimental findings of Fujioka *et al.*, in that the values of this parameter are concentrated above one nucleon mass. Detailed differences between the Monte Carlo and the experimental distributions may be explained by differences between properties of the Monte Carlo and the experimental jets after clarification of the properties of the parameter. Next, after assuming values of the masses and the transverse momenta of the baryons surviving the collision, it was possible to calculate the actual value of the four-momentum transfer between two groups of the produced particles for the Monte Carlo jets. The Hasegawa and Yokoi parameter is found to underestimate this actual value by a factor of about 0.6 ± 0.2, the limits defining the approximate 68% confidence interval for statistical fluctuations of individual measurements about the mean factor 0.6. Finally, the distribution of this actual value itself shows a mean around two nucleon masses, indicating that jets having average properties like the Monte Carlo jets belong to a different class of events from those encompassed by "linked-peripheral" models for which virtual π-meson transfer predominates.

I. INTRODUCTION

In recent years, the trend that the meson shower (jet) particles produced in an ultra-high energy interaction often exhibit spatial groupings has led to the proposal of a model[1–4] in which this generation of particles in groups or "fireballs" is the fundamental production mechanism. A basic quantity for characterizing these interactions is the four-momentum transfer between the groups; this four-momentum transfer is studied here. The importance of this quantity goes beyond its relation to the fireball model because it is a Lorentz invariant quantity, and such quantities are valuable for jet studies where there is difficulty in determining the incident particle's energy.

A parameter for measuring the four-momentum transfer between fireballs was first introduced by Niu in his version of the fireball model. He called this

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[1] P. Ciok, J. Gierula, R. Holynski, A. Jurak, M. Miesowicz *et al.*, Nuovo Cimento **8**, 166 (1958).

[2] G. Cocconi, Phys. Rev. **111**, 1699 (1958).

[3] P. Ciok, T. Coghen, J. Gierula, R. Holynski, A. Jurak *et al.*, Nuovo Cimento **10**, 741 (1958).

[4] K. Niu, Nuovo Cimento **10**, 994 (1958).

parameter the "momentum of interaction" and found it to be distributed around 1 BeV. However, the "momentum of interaction" was the four-momentum transfer between the fireballs for the special case of symmetrical production of the two fireballs in the center-of-mass system, so that the time-like component (energy) of the space-like four-vector describing the momentum transfer was zero. With the observation of asymmetrical emission of particles in the center-of-mass system,[5] the "momentum of interaction" concept was generalized[6] to include the energy component in order to account for this asymmetry. Up to this time, the shower particle groups were identified with not always distinct clusters of points on the log $\tan\theta$ plot,[7] where θ is the laboratory system angle of emission of a shower particle. To avoid the ambiguity of identifying the meson groups and to increase statistical accuracy, Hasegawa and Yokoi[8] recently introduced a more generalized parameter and procedure for determining the four-momentum transfer between two groups of produced particles of a jet. In a given jet, they sysmatically divided the shower particles into two groups; one group consisted of all particles with θ values less than a certain value, the other group consisted of the remaining particles, and all possible such divisions into two groups were made. The Hasegawa and Yokoi parameter, denoted[9] by $|\Delta_l|$ throughout this paper, utilizes θ values and constancy of transverse momenta of the produced particles to determine a deliberate underestimate of the four-momentum transfer between two groups of shower particles. Some of the Japanese collaborators of the International Cooperative Emulsion Flight (ICEF) reported[10] recently an interesting study applying this parameter and procedure to the ICEF jets and comparing the results with predictions of the multiperipheral model of Amati *et al.*[11]

In a recent work,[12] the Monte Carlo method was employed to investigate angular distribution parameters currently in use in analyses of jets. The 3000-BeV Monte Carlo jets described in Ref. 12 were generated from some of the better established experimental information about ultra-high energy interactions and represent average properties of 3000-BeV jets.

In this paper, these Monte Carlo jets are used to study the $|\Delta_l|$ parameter as it was treated in Ref. 10. In Sec. II, the experimental $|\Delta_l|$ distribution of Ref. 10 is compared with the Monte Carlo $|\Delta_l|$ distributions, the purpose being that the comparison may indicate experimental jet properties which can or cannot be explained in terms of properties of the Monte Carlo models. Now, for the Monte Carlo jets, with assumptions about the masses and transverse momenta of the nucleons that survive the collision, there is sufficient information available to calculate exactly from its defining equation the four-momentum transfer between two groups of produced particles; this will be called the "actual four-momentum transfer." In Sec. III is determined a quantitative estimate of the amount by which $|\Delta_l|$ is an underestimate of this actual four-momentum transfer. The investigations in Secs. II and III are in the same spirit as those of Ref. 12. In Sec. IV, a different point of view from Ref. 12 is taken, and the distributions of actual four-momentum transfer of the Monte Carlo jets are themselves presented in order to demonstrate the actual four-momentum transfer that would be expected from jets having average experimental properties.

II. THE FOUR-MOMENTUM TRANSFER PARAMETER

The four-momentum transfer between two groups, 1 and 2, of produced particles of a jet is defined by

$$\delta = (P_{10} - P_1 - \textstyle\sum_1 k_i) = -(P_{20} - P_2 - \textstyle\sum_2 k_j)\,, \quad (1)$$

where P_{10} and P_{20} (P_1 and P_2) are the four-momenta of the incident and target particles before (after) the collision, respectively, and k_i and k_j are the four-momenta of the produced particles of groups 1 and 2, respectively. The point of view taken here is that group 1 is always associated with the incident particle and group 2 with the target particle.

First, a brief review of the derivation[10] of the parameter $|\Delta_l|$ from Eq. (1) is necessary. It is desired to have an underestimate of the magnitude of δ. Taking the longitudinal component of the three-momenta that appear explicitly, assuming (i) the polar angles of emission θ are much less than unity, and omitting the

[5] N. L. Grigorov, V. V. Guseva, N. A. Dobrotin, K. A. Kotelnikov, V. S. Murzin *et al.*, *Proceedings of the Moscow Cosmic Ray Conference* (International Union of Pure and Applied Physics, Moscow, 1960), Vol. 1, p. 143.

[6] E. G. Bubelev, *Proceedings of the Moscow Cosmic Ray Conference* (International Union of Pure and Applied Physics, Moscow, 1960), Vol. 1, p. 285; Z. Koba and A. Krzywicki, Acta Phys. Polon. **21**, 153 (1962); and K. Niu, J. Phys. Soc. Japan **17**, Suppl. A-III, 477 (1962).

[7] The well-known plot of $\tan\theta$ values on the log scale is useful for discussing jet angular distributions because, assuming the velocities of the produced particles in the c.m. system are equal to the velocity of the c.m. system, the distribution of log $\tan\theta$ values about log $(1/\gamma_c)$, where γ_c is the Lorentz factor of the c.m. system, is the logarithmic distribution of the tangents of the polar half-angles in the c.m. system. This plot, however, has the drawback of not presenting azimuthal angle information.

[8] S. Hasegawa and K. Yokoi, Uchusen Kenkyu (Japanese Cosmic Ray Circular) **7**, 186 (1962). This parameter was also reported on behalf of Hasegawa and Yokoi by S. Hayakawa in *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN Scientific Information Service, Geneva, 1962), p. 643.

[9] The subscript l will mean that only the longitudinal (parallel to the direction of the incident-nucleon motion) component of the three-momentum is used where the three-momentum appears explicitly in the expression for the square of the four-momentum transfer. Thus quantities with the subscript l are not strictly *four*-momentum transfers. Also the absolute value symbols will indicate the magnitude of the quantity, i.e., the square root of the sum of the squares of the components.

[10] G. Fujioka, Y. Maeda, O. Minakawa, M. Miyagaki, Y. Tsuzuki *et al.*, Nuovo Cimento, Suppl. (to be published).

[11] D. Amati, S. Fubini, and A. Stanghellini, Nuovo Cimento **26**, 896 (1962).

[12] R. D. Settles and R. W. Huggett, Phys. Rev. **133**, B1305 (1964).

terms relating to the colliding particles, one finds[13]

$$\Delta_l^2=\{\sum_1 p_{t_i}[1+(\mu_i/p_{t_i})^2]\tan\theta_i\}\times\{\sum_2 p_{t_j}[\cot\theta_j+\tfrac{1}{4}(1+[\mu_j/p_{t_j}]^2)\tan\theta_j]\}\,, \quad (2)$$

where p_t is the transverse momentum and μ the mass of each of the produced particles. If (ii) the mean transverse momentum $\langle p_t\rangle$ is approximately equal to 0.4 BeV/c and is independent of θ, if (iii) groups 1 and 2 contain several particles, and if (iv) the charged and neutral particles have similar angular distributions, Eq. (2) reduces to

$$|\Delta_l|=1.5\langle p_t\rangle[(\sum_1 \tan\theta_i)(\sum_2 \cot\theta_j)]^{1/2} \quad (3)$$

after a positive term in each summation is dropped. Equation (3) is the expression for the parameter used in Ref. 10. The assumptions going into this parameter are the statements numbered (i) through (iv) above, (ii) and (iv) being satisfied on the average by the Monte Carlo jets. The parameter is treated here as in Ref. 10, as follows. For each jet, group 1 (group 2) consisted of particles having log $\tan\theta_i$ smaller than (log $\tan\theta_j$ larger than) some dividing point on the log $\tan\theta$ plot; the dividing point was placed successively between adjacent charged-particle log $\tan\theta$ values. Because of assumption (iii) and because of the omission of terms relating to the colliding particles, the two $|\Delta_l|$ values corresponding to the dividing point being between the two largest and two smallest charged-particle log $\tan\theta$ values[14] were considered ambiguous as estimates of $|\delta|$ and were not included in the analysis. (The effect due to the omission of the terms relating to the colliding particles will be clarified in the Appendix.) Thus, there were n_s-3 (n_s=number of shower particles) values of $|\Delta_l|$ per jet. Finally, the approximations that a positive term in each summation of Eq. (2) can be dropped (in order to simplify the expression for $|\Delta_l|$) and that statistical mixing of particles between groups 1 and 2 can be neglected reduce the magnitude of $|\Delta_l|$ and, therefore, are justifiable here since $|\Delta_l|$ is to be an underestimate of $|\delta|$.

Next, a summary of the properties of the Monte Carlo models of Ref. 12 is helpful. Each Monte Carlo jet corresponds to a 3000-BeV nucleon-nucleon interaction in which 16 particles are produced. The 16 produced particles were chosen to be π mesons and K mesons in a random manner, such as to agree on the average with the experimentally determined particle composition of jets, giving an average multiplicity $n_s=9.8$. Four models were used. For the model designated GPTI,[15] each of the first 15 particles was assigned

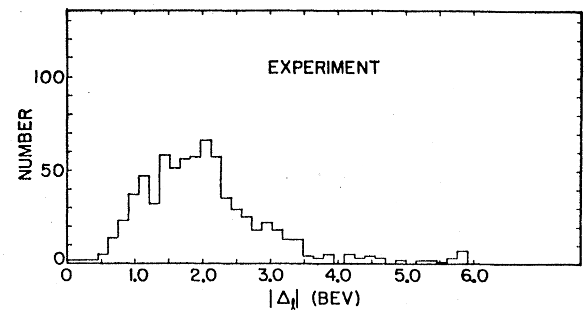


FIG. 1. Distribution of the $|\Delta_l|$ parameter for 70 primary and secondary ICEF jets with $N_h=0,1$ and in the energy range 10^3–10^4 BeV, as reported in Ref. 10. Values of $|\Delta_l|$ corresponding to end log $\tan\theta$ dividing points are not shown.

randomly a transverse momentum from a Gaussian distribution, $G(p_t)dp_t\propto dp_t\exp[-(p_t-\langle p_t\rangle)^2/2\sigma^2]$ with $\langle p_t\rangle=0.37$ BeV/c and $\sigma=0.14$ BeV/c, and was assigned randomly, independently of p_t, a polar angle θ^* in the center-of-mass system from an isotropic distribution. The SPTS model has a skewed p_t distribution, $S(p_t)dp_t\propto dp_tp_t\exp(-p_t/p_0)$ with $p_0=0.19$ BeV/c, and a θ^* distribution of the form $S(\theta^*)d\Omega^*\propto(1/\sin\theta^*)d\Omega^*$ for the first 15 particles (Ω^* is the element of solid angle in the center-of-mass system). Each of the other two models utilizes one property from each of the above two. The SPTI model has the skewed p_t distribution and the isotropic angular distribution, and the GPTS model has the Gaussian p_t distribution and the $(1/\sin\theta^*)d\Omega^*$ angular distribution for the first 15 particles. In all models the azimuthal angles φ of the first 15 particles were picked randomly from 0 to 2π, the p_t, θ^*, and φ values for the 16th particle were determined so as to give a net momentum of zero for the produced particles in the center-of-mass system, and the surviving baryons, not considered among the produced particles, were assumed to carry off with equal and opposite momenta in the center-of-mass system the available energy not taken by the produced particles. The produced particles of all Monte Carlo jets were transformed to the labora-

TABLE I. Characteristics of the experimental and Monte Carlo distributions of the four-momentum transfer parameter $|\Delta_l|$.[a]

	Experiment[b]	Monte Carlo Model GPTI	SPTI	GPTS	SPTS
Excluding values corresponding to end log $\tan\theta$ dividing points					
Mean	2.0	1.63	1.61	1.26	1.27
σ	0.96	0.40	0.40	0.37	0.37
Including values corresponding to end log $\tan\theta$ dividing points					
Mean	1.8	1.47	1.46	1.13	1.14
σ	0.98	0.47	0.47	0.43	0.43
Using the minimum $|\Delta_l|$ value of each jet					
Mean	1.1	1.2	1.2	0.9	0.9
σ	0.42	0.26	0.26	0.25	0.26

[a] All numbers have units of BeV.
[b] From the distribution given in Ref. 10.

[13] Using a similar convention as in Ref. 12 when the distributions of the jet parameters were discussed in general, the lower case letter (δ) specifies the physical quantity and the capital letter (Δ) indicates the parameter that is supposed to be a measure of the physical quantity.

[14] In what follows, such dividing points will be referred to as end log $\tan\theta$ dividing points.

[15] The first three letters specify the p_t distribution, and the last indicates the angular distribution in the c.m. system.

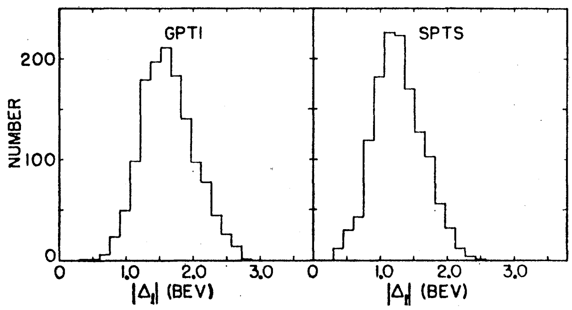


FIG. 2. Distributions of the $|\Delta_l|$ parameter, calculated as in Ref. 10, for jets of the GPTI and SPTS Monte Carlo models. Values of $|\Delta_l|$ corresponding to end log $\tan\theta$ dividing points are not shown. These distributions and that of Fig. 1 are normalized to approximately the same area.

tory system using the exact Lorentz transformation corresponding to a 3000-BeV particle incident upon a nucleon at rest in the laboratory system. Finally, as stated in Ref. 12, of the four, the SPTS model is felt to be the most representative of experimental jets in the 10^3-BeV energy range.

Now, the experimental $|\Delta_l|$ distribution obtained in Ref. 10 is given in Fig. 1, and the Monte Carlo results for the GPTI and SPTS models are presented in Fig. 2. The means and standard deviations σ for all distributions are listed in Table I; also shown in Table I is the effect of including the $|\Delta_l|$ values corresponding to end log $\tan\theta$ dividing points, the effect being to increase the frequency of small $|\Delta_l|$ values.

Before comparing the experimental and Monte Carlo $|\Delta_l|$ distributions, some factors which influence the magnitude of $|\Delta_l|$ should be pointed out. Property (a): The value of $|\Delta_l|$ corresponding to a particular dividing point is approximately proportional to the density of log $\tan\theta$ values in the neighborhood of the dividing point[16]; particles farther than roughly ± 1.0 (logarithmic scale) from the dividing point usually have little effect on the corresponding $|\Delta_l|$ value. Property (b): For a group of jets having similar log $\tan\theta$ distributions, as is the case for the jets of a particular Monte Carlo model, $|\Delta_l|$ is nearly proportional to n_s; taking $|\Delta_l| \propto n_s^{\beta}$, then $\beta = 0.76 \pm 0.04$ for the SPTS Monte Carlo jets. The experimental results of Ref. 10 indicate agreement with this property; using Fig. 6 of Ref. 10, one finds a value of $\beta \approx 0.8 \pm 0.1$. Property (c): The $|\Delta_l|$ values are independent of a translation of a given distribution of log $\tan\theta$ values along the log $\tan\theta$ scale; therefore, $|\Delta_l|$ is independent of γ_c, the Lorentz factor of the center-of-mass system,[17] provided the log $\tan\theta$ distribution is independent of γ_c. Property (c) is evident from the functional form of $|\Delta_l|$; also the experimental results shown in Fig. 7 of Ref. 10 indicate that $|\Delta_l|$ is independent of γ_c.

Finally, comparison of the experimental and Monte Carlo $|\Delta_l|$ distributions shows that both indicate large four-momentum transfers with magnitudes greater than a nucleon mass. This is an important point. The main differences one sees are that the experimental distribution, first, peaks at a higher $|\Delta_l|$ value and, second, has a more pronounced tail extending to high values of $|\Delta_l|$ than the Monte Carlo distributions. These two differences may be explained in terms of properties (a), (b), and (c) above. Because of property (c), the fact that the primary energy of the experimental jets fluctuates while that of the Monte Carlo jets does not is felt to make only a small contribution to these differences. The first difference, the position of the peaks, may be explained by property (b) because the average multiplicity of the experimental jets is 13.4 compared with 9.8 for the Monte Carlo jets. The experimental peak occurs at a $|\Delta_l|$ value roughly 13.4/9.8 times that of the Monte Carlo peaks. The second difference, the experimental high-momentum transfer tail, is more interesting. Such large values of $|\Delta_l|$ arise through properties (a) and (b) from substantial fluctuations in a few events to larger multiplicity and to correspondingly higher densities in some region of the log $\tan\theta$ plot than is possible for the Monte Carlo models.[18] If the Monte Carlo models are taken to represent ultra-high energy elementary collisions, then larger experimental multiplicities and log $\tan\theta$ densities might be explained by secondary interactions occurring inside the target nucleus. Although this effect is mitigated because only events with the number of heavily ionizing tracks $N_h = 0,1$ were used in Ref. 10, it is probably present in a few events nonetheless. However, if the experimental jets are all elementary interactions, higher log $\tan\theta$ densities are possibly due to the presence of stronger correlations among the produced particles of a few jets than exist in the Monte Carlo jets.

The experimental results were compared in Ref. 10 with a distribution of the upper limit of the four-momentum transfer calculated following the multiperipheral model of Amati *et al.*[11] The distribution in Fig. 1 showed large discrepancies with the distribution predicted by the multiperipheral theory if one assumes that the magnitude of the four-momentum transfer is of the order of the π-meson mass. The distribution of the minimum $|\Delta_l|$ value (excluding the $|\Delta_l|$ values corresponding to end log $\tan\theta$ dividing points) for each event was also compared to see if the discrepancies could be reduced, but some differences between theory and experiment remained. The experimental and Monte Carlo distributions of the minimum $|\Delta_l|$ values are compared in Table I. It can be seen that about the same changes occur in going from one Monte Carlo model to

[16] For example, compare the means of the $|\Delta_l|$ distributions for the Monte Carlo models having the isotropic distribution with those having the $(1/\sin\theta^*)d\Omega^*$ distributions; see Table I.

[17] The mean of the log $\tan\theta$ values $\langle \log \tan\theta \rangle$ is equal to $-\log \gamma_c$ in the Castagnoli estimate of γ_c, described in C. Castagnoli, G. Cortini, C. Franzinetti, A. Manfredini, and D. Moreno, Nuovo Cimento **10**, 1539 (1953).

[18] K. Kobayakawa and K. Niu have confirmed (private communications) that the experimental tail is due to a few events with large multiplicity ($n_s \approx 30$).

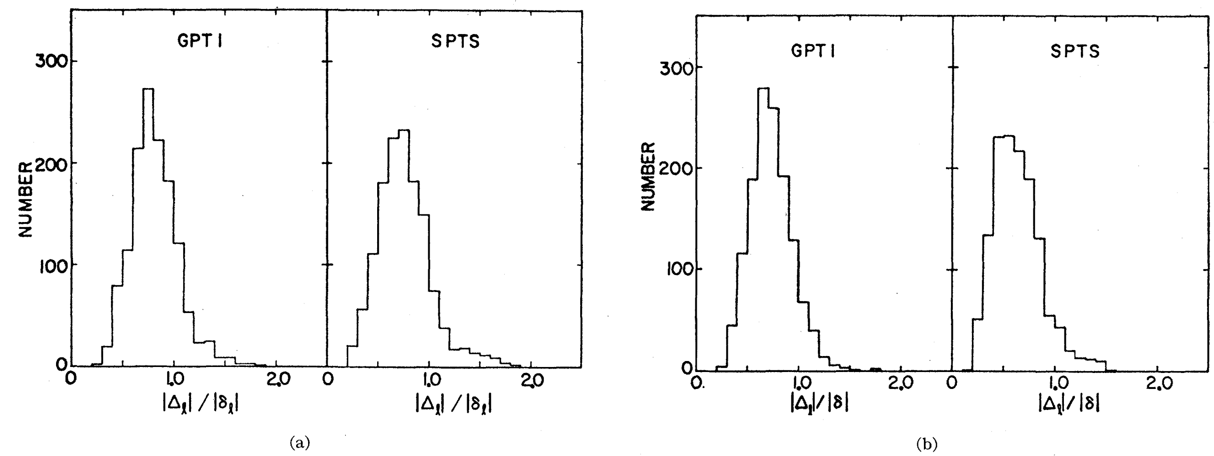


FIG. 3. Typical distributions of the ratio of the parameter $|\Delta_l|$ to the actual four-momentum transfers (a) $|\delta_l|$ and (b) $|\delta|$. The distributions shown are for the GPTI and the SPTS Monte Carlo models. The $|\Delta_l|$ values were calculated using the factor 1.65 in place of 1.5 to correct for the presence of neutral particles, and $|\delta_l|$ and $|\delta|$ were found using $p_{t_n}=0.4$ BeV/c and $M^*=M$. The values of the ratios corresponding to end log tanθ dividing points are not shown.

another as occurred for the distributions of all values of $|\Delta_l|$, and that the experimental distribution, although broader, is not in disagreement with the Monte Carlo results.[19]

III. THE PARAMETER AS A MEASURE OF THE ACTUAL FOUR-MOMENTUM TRANSFER

The next step is to determine how well $|\Delta_l|$ is a measure of the actual four-momentum transfer within the framework of the Monte Carlo models. The center-of-mass data on the Monte Carlo jets plus assumed values of masses M^* and transverse momenta p_{t_n} of the recoil baryons are sufficient information to calculate $|\delta|$ from Eq. (1). For this section, both baryons are assumed to survive with $M^*=M$, the nucleon rest mass, and with $p_{t_n}=0.4$ BeV/c, since these values give fairly representative results when M^* and p_{t_n} are varied. The effects of varying these quantities are small and are demonstrated in Sec. IV. The divisions of the produced particles into two groups were the same as used for the calculation of $|\Delta_l|$ values, the neutral particles being included in the log tanθ plot. Thus, every value of $|\Delta_l|$ has corresponding values of $|\delta_l|$ and $|\delta|$, both of which $|\Delta_l|$ can be considered a measure.[20] Figure 3 presents typical distributions of $|\Delta_l|/|\delta_l|$ and $|\Delta_l|/|\delta|$ for isotropic and $(1/\sin\theta^*)d\Omega^*$ angular distributions in the center-of-mass system, and Table II summarizes the

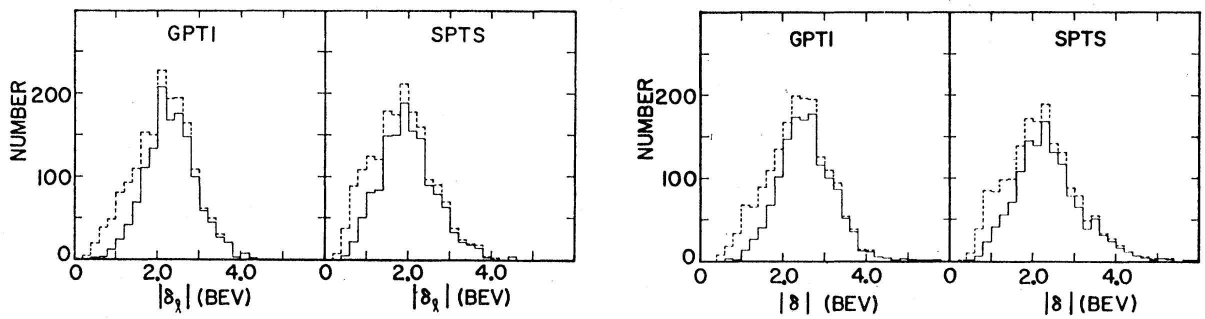
TABLE II. Characteristics of the Monte Carlo distributions of the ratios of the $|\Delta_l|$ parameter[a] to the actual four-momentum transfer,[b] and characteristics of the Monte Carlo distributions of actual four-momentum transfer.[c]

Distribution		Monte Carlo model GPTI	SPTI	GPTS	SPTS
		Excluding values corresponding to end log tanθ dividing points			
$\|\Delta_l\|/\|\delta_l\|$	Mean	0.81	0.74	0.83	0.76
	σ	0.23	0.24	0.24	0.26
$\|\Delta_l\|/\|\delta\|$	Mean	0.73	0.67	0.70	0.64
	σ	0.21	0.22	0.22	0.23
		Including values corresponding to end log tanθ dividing points			
$\|\Delta_l\|/\|\delta_l\|$	Mean	0.80	0.73	0.80	0.74
	σ	0.25	0.27	0.26	0.28
$\|\Delta_l\|/\|\delta\|$	Mean	0.72	0.65	0.67	0.62
	σ	0.23	0.25	0.25	0.25
		Excluding values corresponding to end log tanθ dividing points			
$\|\delta_l\|$	Mean	2.3	2.5	1.8	2.0
	σ	0.60	0.71	0.54	0.65
$\|\delta\|$	Mean	2.6	2.9	2.1	2.4
	σ	0.66	0.82	0.64	0.79
		Including values corresponding to end log tanθ dividing points			
$\|\delta_l\|$	Mean	2.1	2.4	1.6	1.8
	σ	0.70	0.81	0.60	0.72
$\|\delta\|$	Mean	2.4	2.7	2.0	2.2
	σ	0.76	0.91	0.71	0.86

[a] The $|\Delta_l|$ parameter for this table was calculated using 1.65 instead of 1.5 to correct for the presence of neutral particles.
[b] The actual four-momentum transfer for this table was calculated assuming the baryons survive the collision with masses equal to the nucleon rest mass and with transverse momenta equal to 0.4 BeV/c.
[c] All numbers for the actual four-momentum transfer distributions have units of BeV.

[19] Since Fig. 1 includes both primary and secondary jets, the experimental distribution represents an admixture of meson- and nucleon-initiated events. However, the properties built into the Monte Carlo jets were observed mainly in nucleon-nucleon interactions. Within the accuracy of cosmic ray experiments, there appears to be no evidence for differences which might exist between properties of meson- versus nucleon-initiated jets beyond the kinematical ones. Also K. Kobayakawa, using another four-momentum transfer parameter (see Ref. 24), found no difference in the four-momentum transfer properties of primary and secondary jets.

[20] If log tanθ values for neutral particles fall between the log tanθ values for two charged particles, then there are several $|\delta|$ values corresponding to the $|\Delta_l|$ for which the dividing point falls between the log tanθ values for the two charged particles. The convention followed here was that the $|\Delta_l|$ corresponding to the dividing point lying between the log tanθ values for the two charged particles was associated with the $|\delta|$ corresponding to the dividing point being between the larger charged-particle log tanθ value and the next smaller neutral-particle log tanθ value.



(a) (b)

FIG. 4. Typical distributions of the actual four-momentum transfers (a) $|\delta_l|$ and (b) $|\delta|$. The distributions shown are for the SPTS and the GPTI Monte Carlo models. The $|\delta_l|$ and $|\delta|$ values for these figures were calculated using $p_{t_n}=0.4$ BeV/c and $M^*=M$. Solid line: distribution excluding values corresponding to end log $\tan\theta$ dividing points. Dashed line: distribution including values corresponding to end log $\tan\theta$ dividing points.

distributions of $|\Delta_l|/|\delta_l|$ and $|\Delta_l|/|\delta|$ for all of the Monte Carlo models. Since 1.65 is the proper average factor to correct for the presence of neutral particles in the Monte Carlo jets (nonpions were included in the produced particles), 1.65 was used instead of 1.5 in computing $|\Delta_l|$ for these results. The means of the distributions for the GPTI model are larger than those for the SPTI model by approximately the inverse of the ratios of the means of the p_t distributions of these respective models (similarly for the GPTS and the SPTS models). Table II shows that $|\Delta_l|$ as a measure of the actual four-momentum transfer is rather independent of the angular distribution and that the $|\Delta_l|/|\delta|$ values corresponding to end log $\tan\theta$ dividing points, while they broaden the distributions slightly, do not affect largely the mean value of the ratio $|\Delta_l|/|\delta|$.

IV. THE ACTUAL FOUR-MOMENTUM TRANSFER

The distribution of the actual four-momentum transfer for the Monte Carlo jets is of interest because it is the distribution of four-momentum transfer between two groups of particles of jets possessing some average experimental properties of ultra-high energy interactions. Table II gives the characteristics of the distributions of $|\delta_l|$ and $|\delta|$, calculated as described in Sec. III for dividing points between adjacent charged-particle log $\tan\theta$ values. The effect of including $|\delta_l|$ and $|\delta|$ values corresponding to end log $\tan\theta$ dividing points is shown and is seen to be similar to the effect on the $|\Delta_l|$ distributions of including $|\Delta_l|$ values corresponding to end log $\tan\theta$ dividing points (Table I). Figure 4 demonstrates typical $|\delta_l|$ and $|\delta|$ distributions, the values corresponding to end log $\tan\theta$ dividing points being indicated by the dashed curve. For the remainder of this section, the actual four-momentum transfer distributions will include values corresponding to end log $\tan\theta$ dividing points because these values have physical meaning in Eq. (1).

The effects of variation of the transverse momenta[21] p_{t_n} and masses $M^*=\alpha M$ of the recoil baryons on various actual four-momentum transfer distributions are demonstrated in Table III. Two Monte Carlo models, GPTI and SPTS, are sufficient to show these effects.

In addition to $|\delta_l|$ and $|\delta|$, the following momentum-transfer quantities are included for completeness. The $|\boldsymbol{\delta}_t|$ distribution is that of the transverse component of three-momentum transfer; for an individual jet, one has $|\boldsymbol{\delta}_t|^2=|\delta|^2-|\delta_l|^2$. The quantity $|\delta|^{\text{c.m.}}$ is the actual four-momentum transfer corresponding to the dividing point being the natural division between forward and backward directions in the center-of-mass system.[22]

TABLE III. Effects of variation of the transverse momenta and the masses of the recoil baryons on various actual four-momentum transfer distributions of the GPTI and SPTS Monte Carlo models.[a]

			GPTI			SPTS		
Quantity	α		$p_{t_n}=0.0$	0.4	1.0	$p_{t_n}=0.0$	0.4	1.0
$\|\delta_l\|$	1.0	Mean	2.1	2.1	2.2	1.8	1.8	1.9
		σ	0.70	0.70	0.69	0.73	0.72	0.69
	1.5	Mean	2.2	2.2	2.2	1.9	1.9	1.9
		σ	0.68	0.68	0.67	0.69	0.69	0.67
	2.0	Mean	2.2	2.2	2.2	2.0	2.0	2.0
		σ	0.67	0.66	0.66	0.66	0.66	0.65
$\|\delta\|$	1.0	Mean	2.3	2.4	2.6	2.2	2.2	2.4
		σ	0.77	0.76	0.71	0.86	0.86	0.83
$\|\boldsymbol{\delta}_t\|$		Mean	0.85	0.93	1.28	1.05	1.11	1.41
		σ	0.53	0.54	0.59	0.70	0.71	0.76
$\|\delta\|^{\text{c.m.}}$	1.0	Mean	2.7	2.7	2.9	2.4	2.4	2.6
		σ	0.55	0.56	0.58	0.66	0.66	0.69
$\|\delta'\|$	1.0	Mean	0.19	0.49	1.1	0.52	0.77	1.5
		σ	0.22	0.22	0.28	0.56	0.56	0.71

[a] All distributions in this table include values corresponding to end log $\tan\theta$ dividing points except the last two distributions, for which only one four-momentum transfer per jet is defined. All numbers have units of BeV.

[21] The experimental results of Chong Oh Kim, Ph.D. thesis, University of Chicago, 1963 (Phys. Rev., to be published), indicate a reasonable range for p_{t_n} values to be from 0.0 to 1.0 BeV/c. Dr. Kim, on the basis of a few observations in interactions in the 10^3-BeV energy range, found $\langle p_{t_n}\rangle=0.35$ BeV/c.

[22] For the Monte Carlo jets, p_t is independent of θ^*, and the exact Lorentz transformation to the laboratory system often caused the ordering of the particles by θ^* values to be different from the ordering by log $\tan\theta$ values. Therefore, there may not be a dividing point used for the calculation of $|\delta|$ which corresponds to that used for the calculation of $|\delta|^{\text{c.m.}}$.

Finally, $|\delta'|$ is the four-momentum transfer to the colliding nucleons in the Monte Carlo models and is defined by the equation

$$|\delta'|^2=(P_{10}-P_1)^2=(P_{20}-P_2)^2. \qquad (4)$$

The values of $|\delta'|$ for incident and target nucleons are the same in the Monte Carlo models because before or after the collision the baryons have symmetrical properties.

The variation of $|\delta|$ and $|\delta|^{\text{c.m.}}$ with the surviving baryon mass is similar to that of $|\delta_l|$, so only that of $|\delta_l|$ is given. As seen in Table III, the means of the $|\delta_l|$, $|\delta|$, and $|\delta|^{\text{c.m.}}$ distributions increase and their standard deviations σ decrease as p_{t_n} and $\alpha\equiv M^*/M$ increase, but the effect is not a large one. Table III also shows that $|\delta|^{\text{c.m.}}$, for which there is symmetry between the two groups of produced particles (i.e., on the average the two groups have the same properties in the center-of-mass system), gives larger values of actual four-momentum transfer than $|\delta|$, for which on the average there is not this symmetry. The four-momentum transfer between the two groups is smaller for the symmetrical case than for the nonsymmetrical case only for jets in which the groups are well separated on the log tanθ plot; this applies to both $|\Delta_l|$ and $|\delta|$.

The $|\boldsymbol{\delta}_t|$ distribution is independent of M^*. The mean[23] of the $|\boldsymbol{\delta}_t|$ distribution is fairly sensitive to p_{t_n}, as might be expected.

The dependence of $|\delta'|$ on p_{t_n} and M^* can be inferred from Eq. (4) which, when evaluated in the center-of-mass system, becomes

$$|\delta'|^2=-M^2-M^{*2}-2\{p_{20}[E_2^2-p_{t_n}^2-M^{*2}]^{1/2}-E_{20}E_2\}, \qquad (5)$$

where p_{20} and E_{20} are the momentum and energy of the target nucleon before the collision and E_2 is the energy of the target baryon after the collision. Table III and Eq. (5) indicate that $|\delta'|$ is sensitive to p_{t_n}, M^*, and the Monte Carlo model used. The sensitivity of $|\delta'|$ to the latter is due to the dependence of E_2 on the energy used in each jet for particle production.

Finally, comparison of results in Table III for the isotropic and the $(1/\sin\theta^*)d\Omega^*$ angular distributions indicates that property (a) of $|\Delta_l|$, given in Sec. II, is also valid qualitatively for $|\delta_l|$, $|\delta|$, and $|\delta|^{\text{c.m.}}$. However, property (b) of $|\Delta_l|$ is not valid for $|\delta_l|$ within the framework of the Monte Carlo models. In Sec. II, it was stated that $|\Delta_l|\propto n_s^{0.8}$. In contrast, $|\delta_l|$ shows little dependence on n_s. Because both charged and neutral particles were used to calculate $|\delta_l|$, this difference is explained by the fact that the total number of particles was the same for all Monte Carlo jets.

V. SUMMARY AND CONCLUSIONS

In Sec. II experimental and Monte Carlo $|\Delta_l|$ distributions are found to agree in that they both indicate a momentum transfer larger than one nucleon mass. The fact that the experimental distribution peaks at a higher $|\Delta_l|$ value than the Monte Carlo distributions is attributed to the difference between experimental and Monte Carlo mean multiplicities. The tail of large $|\Delta_l|$ values in the experimental distribution is of interest because it does not have a Monte Carlo counterpart. This tail is due in a few events to fluctuations to large multiplicity and to corresponding high density in regions of the log tanθ plot. Such fluctuations may arise from the occurrence of secondary interactions inside the target nucleus in a few experimental jets or from the presence of stronger correlations among the produced particles of a few experimental jets than are present in the Monte Carlo jets.

In Sec. III the $|\Delta_l|$ parameter can be seen from Table II to be an underestimate of the actual four-momentum transfer $|\delta|$ by a factor of about 0.6 ± 0.2 (using results from the SPTS model), the limits defining the approximate 68% confidence interval for statistical fluctuations of individual measurements about 0.6. This result is fairly independent of whether or not values corresponding to end log tanθ dividing points are included.

The results of Sec. IV (see Tables II and III and Fig. 4) indicate that, rather independently of the masses and transverse momenta of the recoil baryons, the average actual four-momentum transfer characterizing the Monte Carlo models is roughly 2 BeV/c. Therefore, insofar as the properties of the Monte Carlo models represent the average properties of experimental jets, small four-momentum transfer between two groups of the produced particles of the order of a π-meson mass is not a dominant mechanism by which jet producing interactions take place.

This last conclusion is the same as that which is drawn from the ICEF experimental results[24] in Ref. 10. Thus, one may conclude that ultra-high energy interactions represented by the Monte Carlo models and by the ICEF jets are a different class of events from those discussed in theoretical studies of very-high-energy "linked-peripheral" interactions in which the magni-

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[23] A parameter Δ_t for estimating $|\boldsymbol{\delta}_t|$ is given by S. Hayakawa in *Proceedings of the Trieste Seminar on Theoretical Physics, July-August, 1962* (IAEA, Vienna, 1963), p. 485 along with a complete discussion of four-momentum transfer and its relation to the fireball models. This parameter is $\Delta_t^2\approx n_1n_2\langle p_t\rangle^2$ where n_1 and n_2 are the number of particles in groups 1 and 2, respectively. However, even neglecting the presence of neutral particles, for $n_s=10$ this yields a mean value of Δ_t of approximately 1.7 BeV, a definite overestimate of $|\boldsymbol{\delta}_t|$ (see Table III). This overestimation of $|\boldsymbol{\delta}_t|$ is due to the fact that the derivation of the expression for Δ_t^2 apparently did not take into account the requirement that the transverse momenta of the particles should be summed like vectors instead of like scalars.

[24] Similar results using different experimental approaches from the $|\Delta_l|$ parameter were found by K. Niu, Ref. 4, using his "momentum of interaction"; K. Imaeda and M. Kazuno, Nuovo Cimento **27**, 119 (1963), using also a "momentum-of-interaction" parameter; and K. Kobayakawa, Suppl. Progr. Theoret. Phys. (to be published), using a parameter proposed by himself and K. Nishikawa.

tude of the four-momentum transfer is assumed to be of the order of a virtual π-meson mass.[25] This is, of course, provided that the method of the $|\Delta_l|$ parameter as used in Ref. 10 and the "linked-peripheral" models are implying similar interpretations for the interaction and therefore should be compared.[26]

In conclusion, the $|\Delta_l|$ parameter is a measure of a Lorentz-invariant quantity, making it very valuable for studies of ultra-high energy interactions. It has fairly good properties, and its careful use may uncover interesting features of jets. For example, a possible indirect use is that a determination of the experimental $|\Delta_l|$ versus γ_c dependence for reasonably fixed n_s might indicate how the presence of correlations in experimental jets changes with energy.

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APPENDIX

The effect of neglecting terms relating to the colliding particles in the derivation of Eq. (3) needs clarification. It is desired to know the conditions under which neglecting these terms could cause $|\Delta_l|$ to be an overestimate of $|\delta|$. There are two cases to consider: The surviving collision partners (1) do not and (2) do appear in the log $\tan\theta$ plot.

In case (1), omission of the terms relating to the incident particle always causes $|\Delta_l|$ to be an underestimate of $|\delta|$. The same is true for omission of the target particle terms if the recoil target particle is emitted forward in the laboratory system (i.e., if $\theta_2 < 90°$, where θ_2 is the recoil-target-particle emission angle). However, if the recoil target particle is emitted backward in the laboratory system ($\theta_2 > 90°$), omission of the target-particle terms allows $|\Delta_l|$ to be an underestimate of $|\delta|$ only if $(E_2 - M)|\tan\theta_2| > p_{t_2}$, where M is the target-particle rest mass, and E_2 and p_{t_2} are the energy and transverse momentum of the recoil target particle.

In case (2), omission of the terms relating to the incident particle always causes $|\Delta_l|$ to be an underestimate of $|\delta|$, and neglecting the terms relating to the target particle allows $|\Delta_l|$ to be an underestimate of $|\delta|$ only if $(E_2 - M)\tan\theta_2 > p_{t_2}$. These statements pertaining to case (2) apply whenever the log $\tan\theta$ value of the recoil incident particle is in group 1 and the log $\tan\theta$ value for the recoil target particle is in group 2. If, for some dividing point, the log $\tan\theta$ value of the recoil incident (target) particle falls in group 2 (group 1), no simple statement as to when $|\Delta_l|$ is, or is not, an underestimate of $|\delta|$ is possible. If the log $\tan\theta$ values for charged surviving collision partners occur on the ends of the log $\tan\theta$ plot, then the log $\tan\theta$ value of the recoil incident (target) particle is in group 1 (group 2) for all dividing points, and the opening statement of this paragraph pertaining to case (2) is valid for all dividing points.

The effects due to omission of terms relating to the collision partners have the most influence on how well $|\Delta_l|$ is an estimate of $|\delta|$ for end log $\tan\theta$ dividing points, and this is one reason that $|\Delta_l|$ values corresponding to end log $\tan\theta$ dividing points were not considered in the analysis in Ref. 10. However, one can see from Table II that the values corresponding to end log $\tan\theta$ dividing points can be included in an analysis without seriously affecting $|\Delta_l|$ as a measure of $|\delta_l|$ and $|\delta|$. [The Monte Carlo jets correspond to case (1) because the surviving baryons were not considered among the produced particles for the Monte Carlo models.] Also apparent from Table II is that the effects of approximations tending to reduce the magnitude of $|\Delta_l|$ dominate, so that on the average $|\Delta_l|$ is an underestimate of $|\delta_l|$ and $|\delta|$.

[25] Examples of "linked-peripheral" models are found in Amati *et al.*, Ref. 11, and F. Salzman, Phys. Rev. **131**, 1786 (1963).

[26] In this respect, it would be interesting to see the method of the $|\Delta_l|$ parameter applied to the phenomenological (Monte Carlo) model for peripheral interactions of O. Czyzewski and A. Krzywicki, Nuovo Cimento **30**, 603 (1963) and to the two-fireball Monte Carlo jets of S. Alper and E. M. Friedländer, Rev. Phys., Acad. Rep. Populaire Roumaine **7**, 311 (1962).